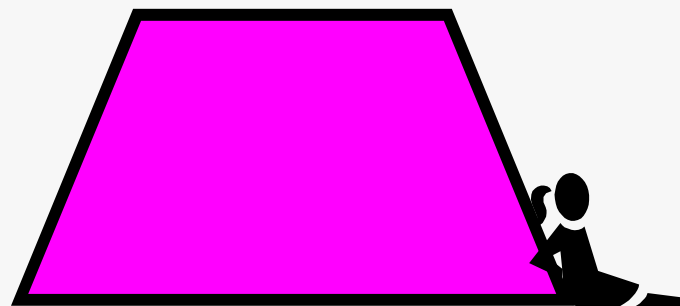


Gdje se krije poučak o kosinusu u prodajnom centru?

Prof. Dobrila Gotovac Stipaničev
I. Gimnazija - Split



Malo povijesti

Iako poučak o kosinusu nije bio tada poznat, Euklidovi Elementi (3.st. prije Krista), sadrže geometrijski teorem koji mu je gotovo ekvivalentan.

Teorem u 10.st. Spominju i Arapski matematičari, a popularizirao ga je Francoise Viete u 16.st. Početkom 19.st. kosinuso

poučak poprima današnji oblik.

Malo povijesti

Tako se na primjer u knjizi '**Plane and spherical trigonometry, surveying and tables**' autora George Albert Wentworta iz 1895. g. poučak o kosinusu pojavljuje točno u obliku kakav i danas koristimo.

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TRIGONOMETRY.

§ 34. LAW OF COSINES.

This law gives the value of one side of a triangle in terms of the other two sides and the angle included between them.

In Figs. 31 and 32, $a^2 = b^2 + c^2 - 2bc \cos A$.

In Fig. 31, $BD = c - AD$;

in Fig. 32, $BD = AD - c$;

in both cases, $BD^2 = AD^2 - 2c \times AD + c^2$.

Therefore, in all cases, $a^2 = b^2 + AD^2 + c^2 - 2c \times AD$.

Now, $b^2 + AD^2 = b^2$,

and $AD = b \cos A$.

Therefore, $a^2 = b^2 + c^2 - 2bc \cos A$.

[26]

In like manner, it may be proved that

$$b^2 = a^2 + c^2 - 2ac \cos B,$$

$$c^2 = a^2 + b^2 - 2ab \cos C.$$

$$a^2 = b^2 + c^2 - 2bc \cos \alpha$$

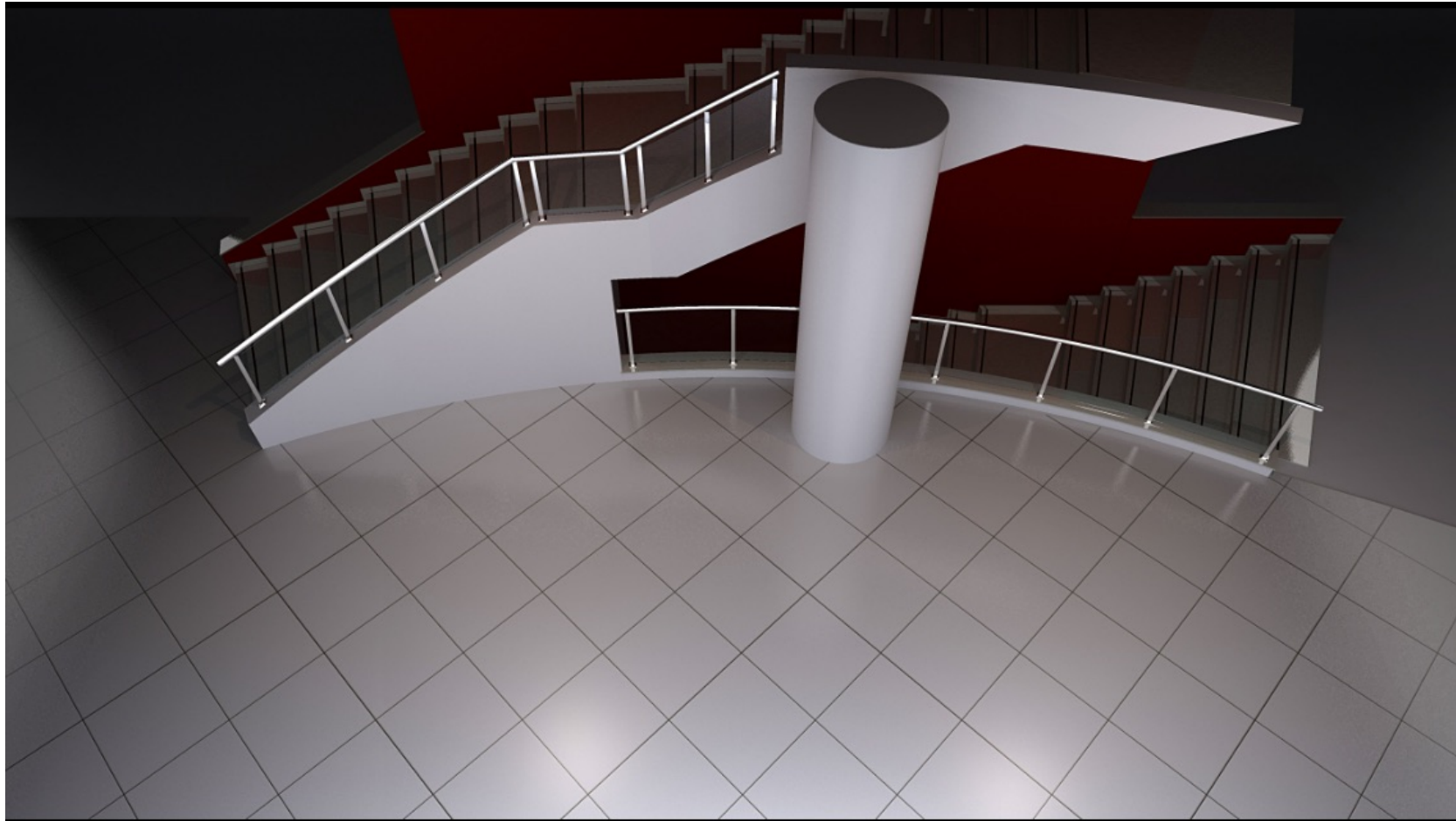
$$b^2 = a^2 + c^2 - 2ac \cos \beta$$

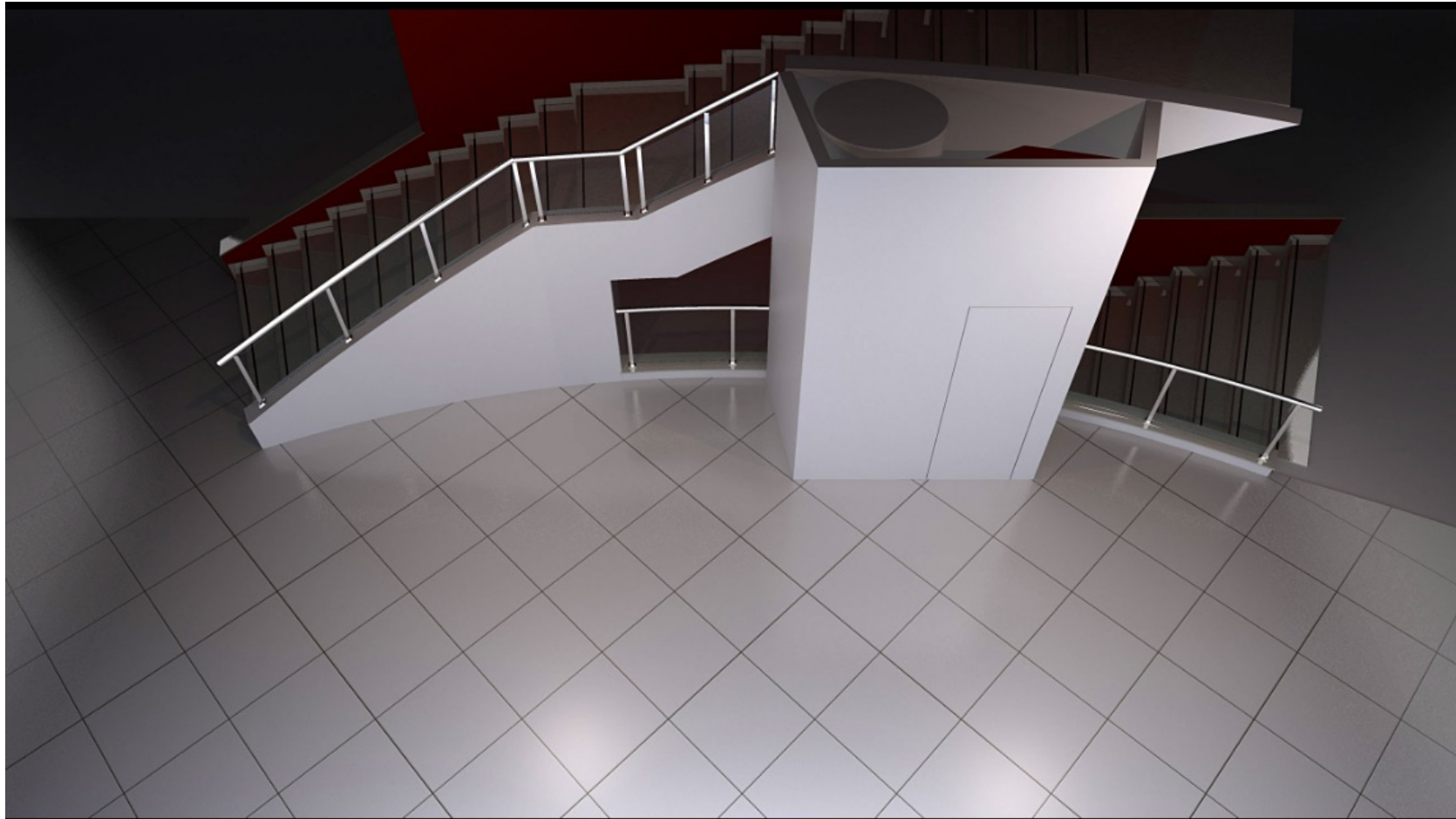
$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$

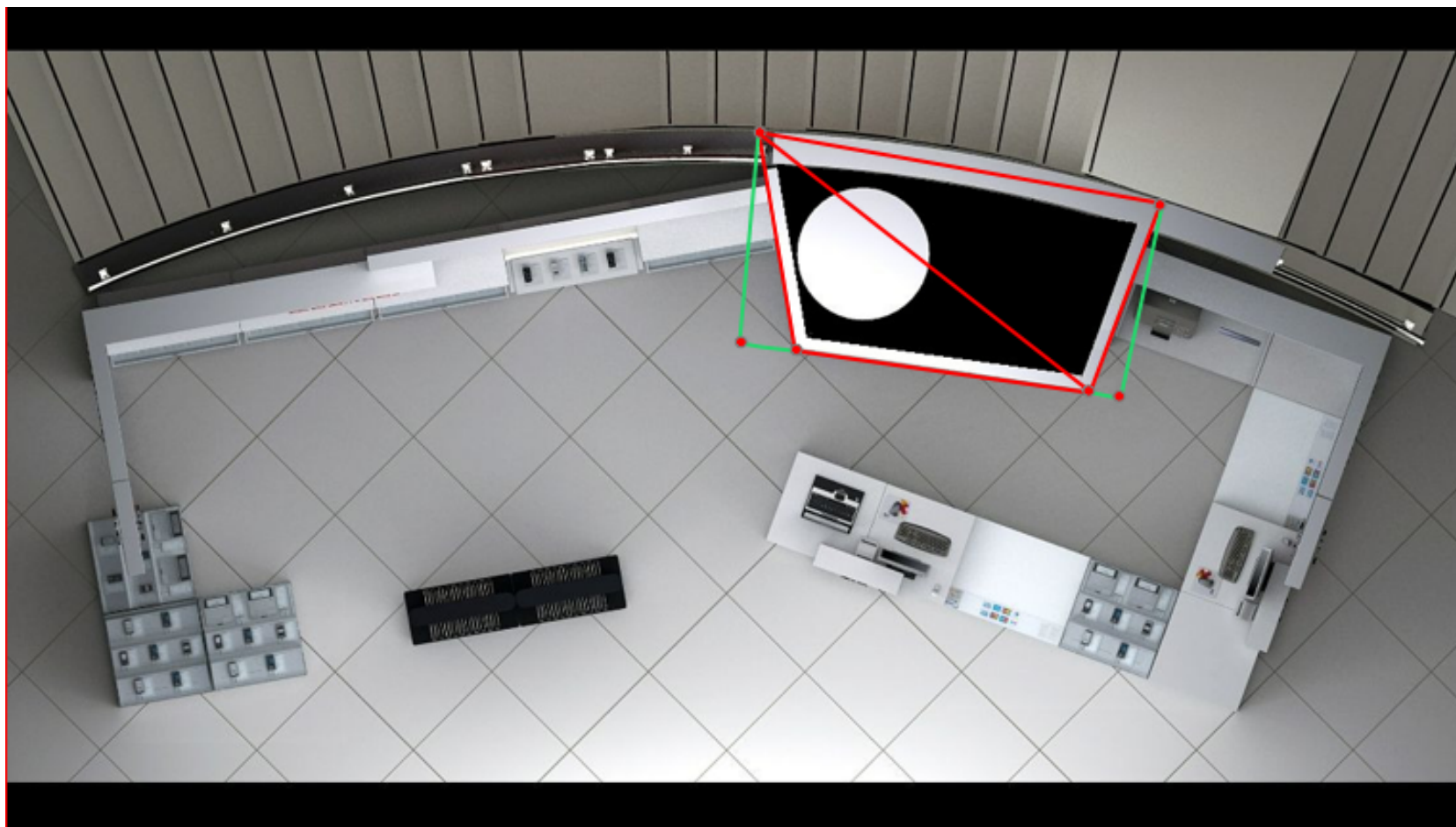
Priča

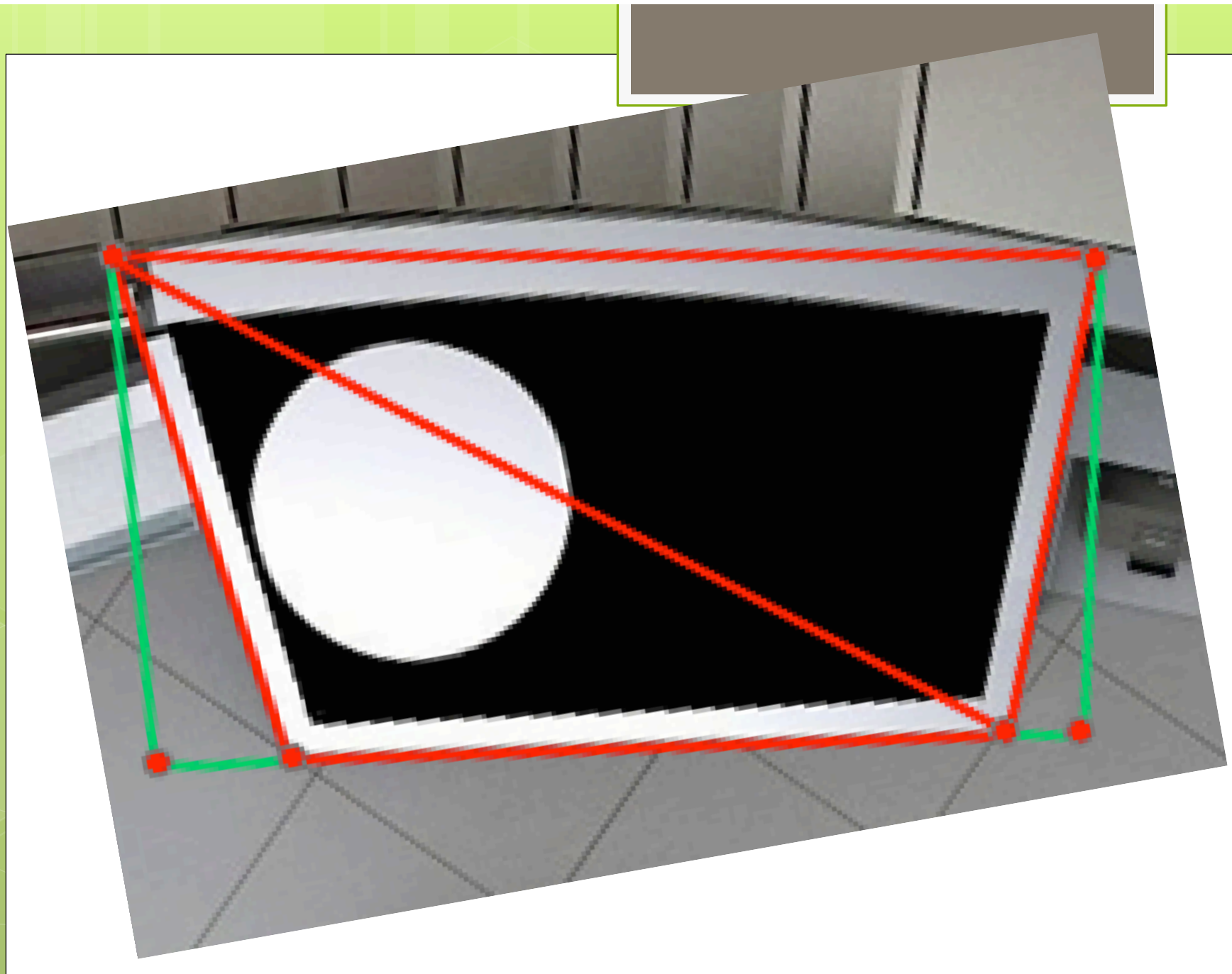
Moja priča počinje jedne večeri kada mi je zazvonio mobitel. Prijatelj je tražio moju pomoć. Dobio je zadatak rekonstruirati prodajni štand po zadanom nacrtu.

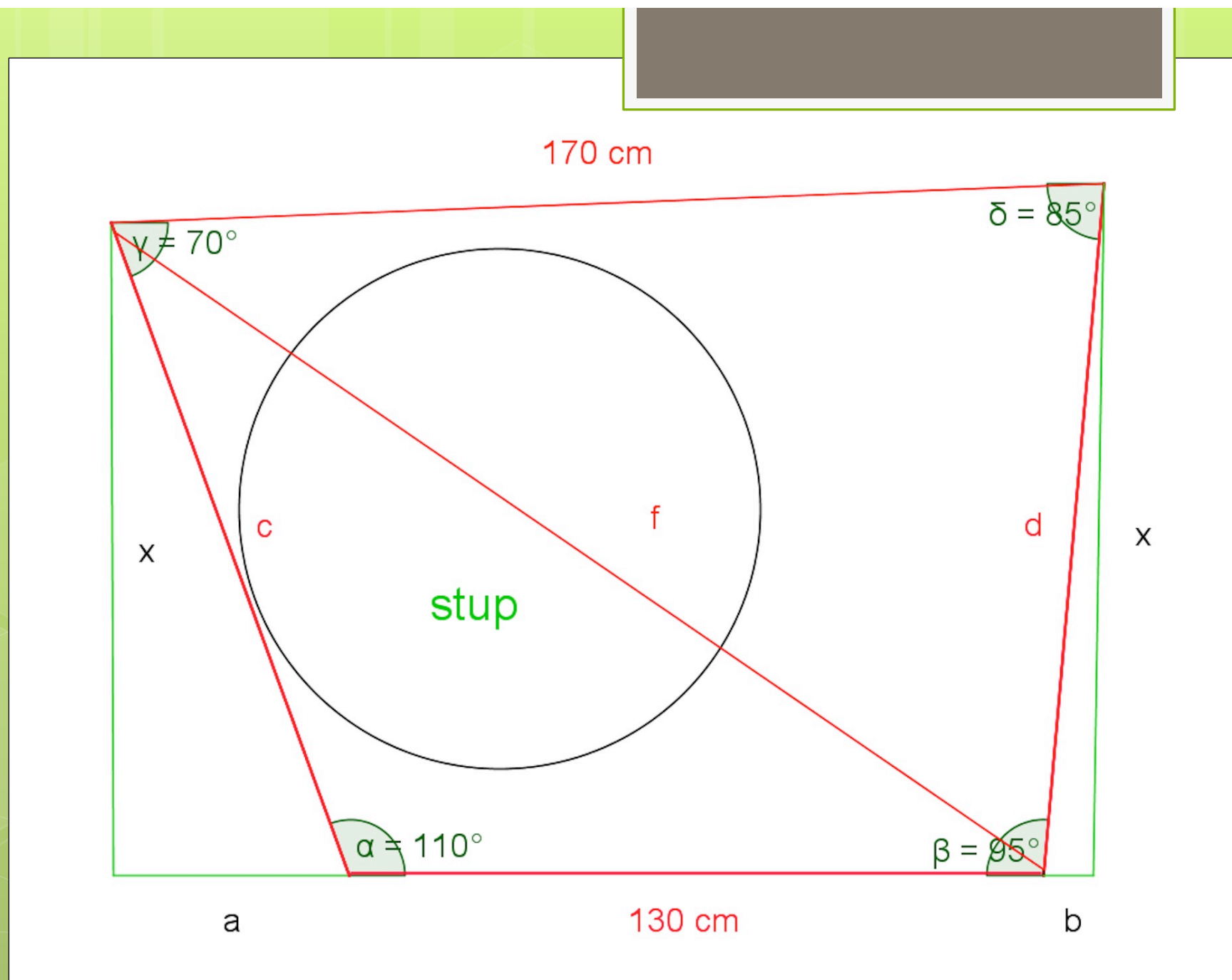
Budući da mu u srednjoj školi matematika nije bila omiljeni predmet, nije se ni mogao sjetiti da mu problem može riješiti poučak o kosinusu.











$$\sin 70^\circ = \frac{x}{c} \Rightarrow c = \frac{x}{\sin 70^\circ} \quad c = 94,29 \text{ cm}$$

$$\sin 85^\circ = \frac{x}{d} \Rightarrow d = \frac{x}{\sin 85^\circ} \quad d = 88,94 \text{ cm}$$

$$f^2 = d^2 + 170^2 - 2 \cdot 170 \cdot d \cdot \cos 85^\circ$$

$$f^2 = c^2 + 130^2 - 2 \cdot 130 \cdot c \cdot \cos 110^\circ$$

$$\frac{x^2}{\sin^2 85^\circ} + 170^2 - 340 \cdot \frac{x}{\sin 85^\circ} \cdot \cos 85^\circ = \frac{x^2}{\sin^2 70^\circ} + 130^2 - 260 \cdot \frac{x}{\sin 70^\circ} \cdot \cos 110^\circ$$

$$1,00765x^2 + 28900 - 29,7461x = 1,13247x^2 + 16900 + 94,6322x$$

$$-0,1248x^2 - 124,38x + 12000 = 0$$

$$x_{1/2} = \frac{124,378 \pm \sqrt{(-124,38)^2 + 4 \cdot 0,1248 \cdot 12000}}{-2 \cdot 0,1248}$$

$$x_1 = -1085,063 \text{ cm} \quad x_2 = 88,602 \text{ cm}$$

Primjeri primjene poučka o sinusu i kosinusu u prodajnom centru:

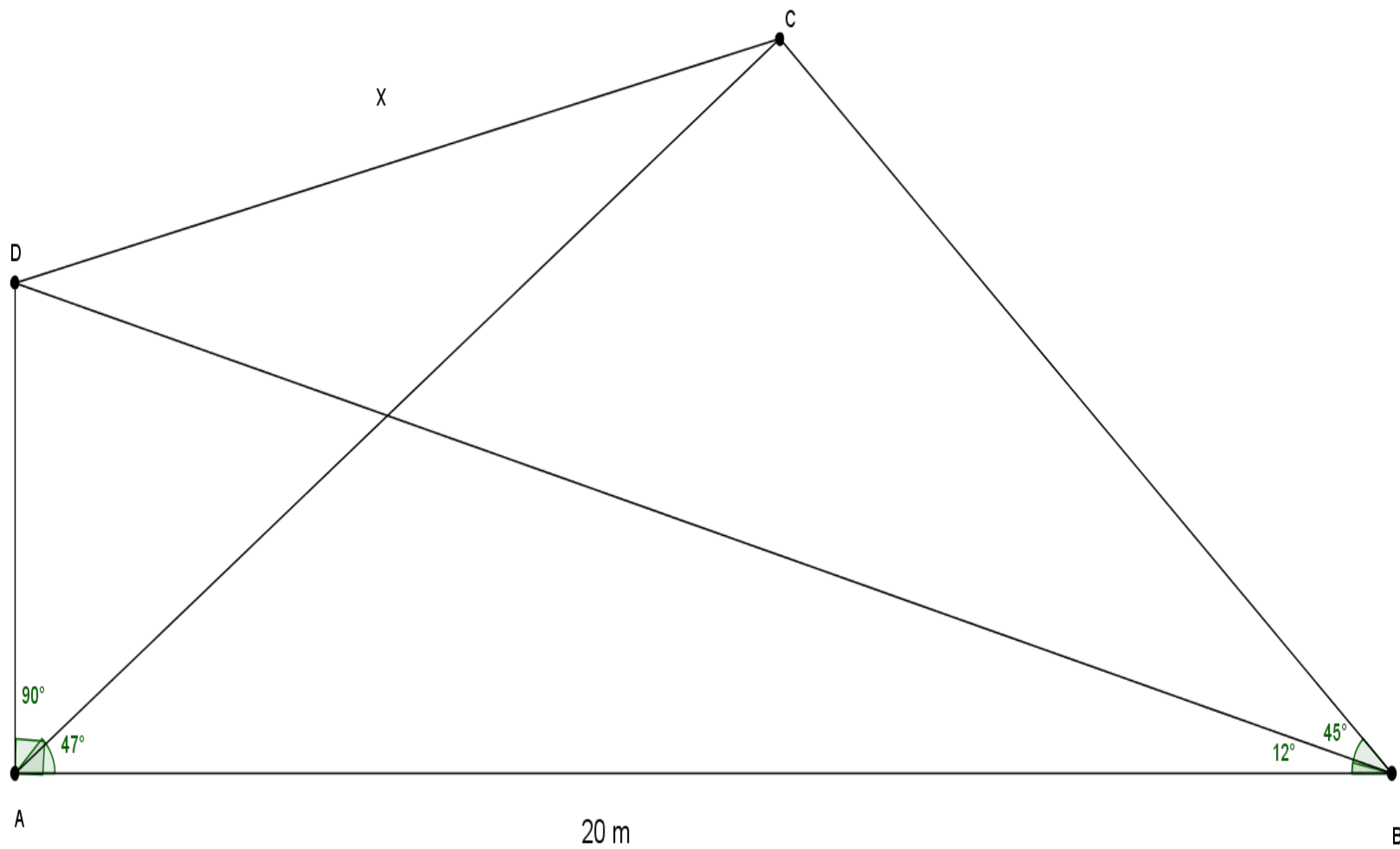
Primjer 1: Žele i Jure

Žele i Jure uživaju u popodnevnoj kavici u jednom našem prodajnom centru.

U jednom trenutku Žele primijeti svog prijatelja Juru na drugom kraju kafića i odmah ga počne dozivati.

Koliki put prevali zvuk od Žele do Jure?

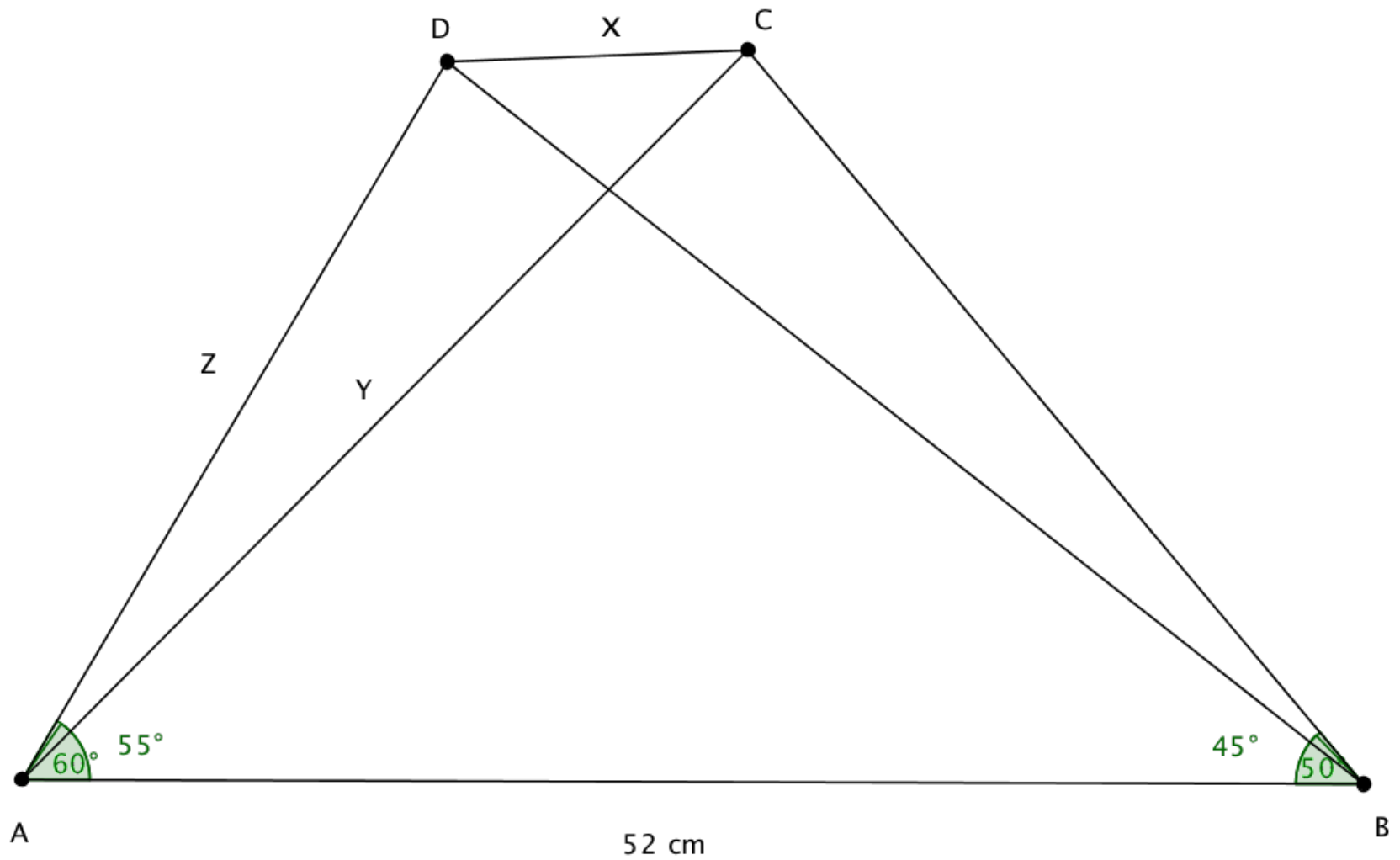




Primjer 2: Lutka

Lutka u izlogu prodavaonice postavljena je tako da joj koljena sa vrhovima cipela formiraju trapez. Ako je poznata udaljenost između vrhova cipela i kutovi između te dužine i spojnice vrhova i koljena odredi na kojoj su međusobnoj udaljenosti koljena.





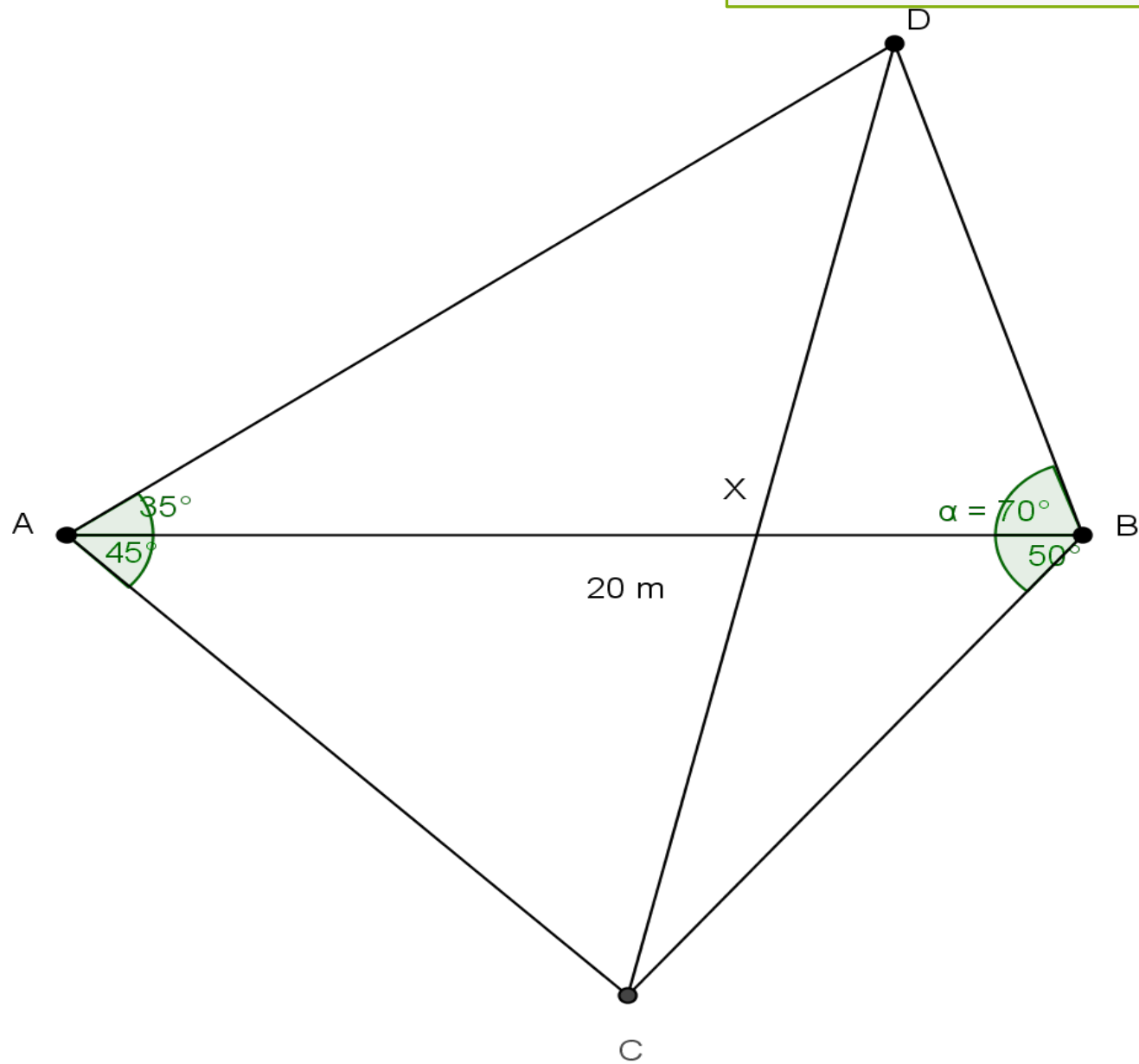
Primjer 3: Dekoracija

Dekorateri su dobili zadatak da povodom nadolazećih blagdana postave prigodnu dekoraciju. Traka na kojoj je pričvršćena dekoracija mora biti fiksirana na konstrukciju staklenog krova, tako da povezuje vrhove pravokutnika na suprotnim stranama sljemena, koji imaju zajednički vrh na njemu.

Kako će odrediti duljinu trake?

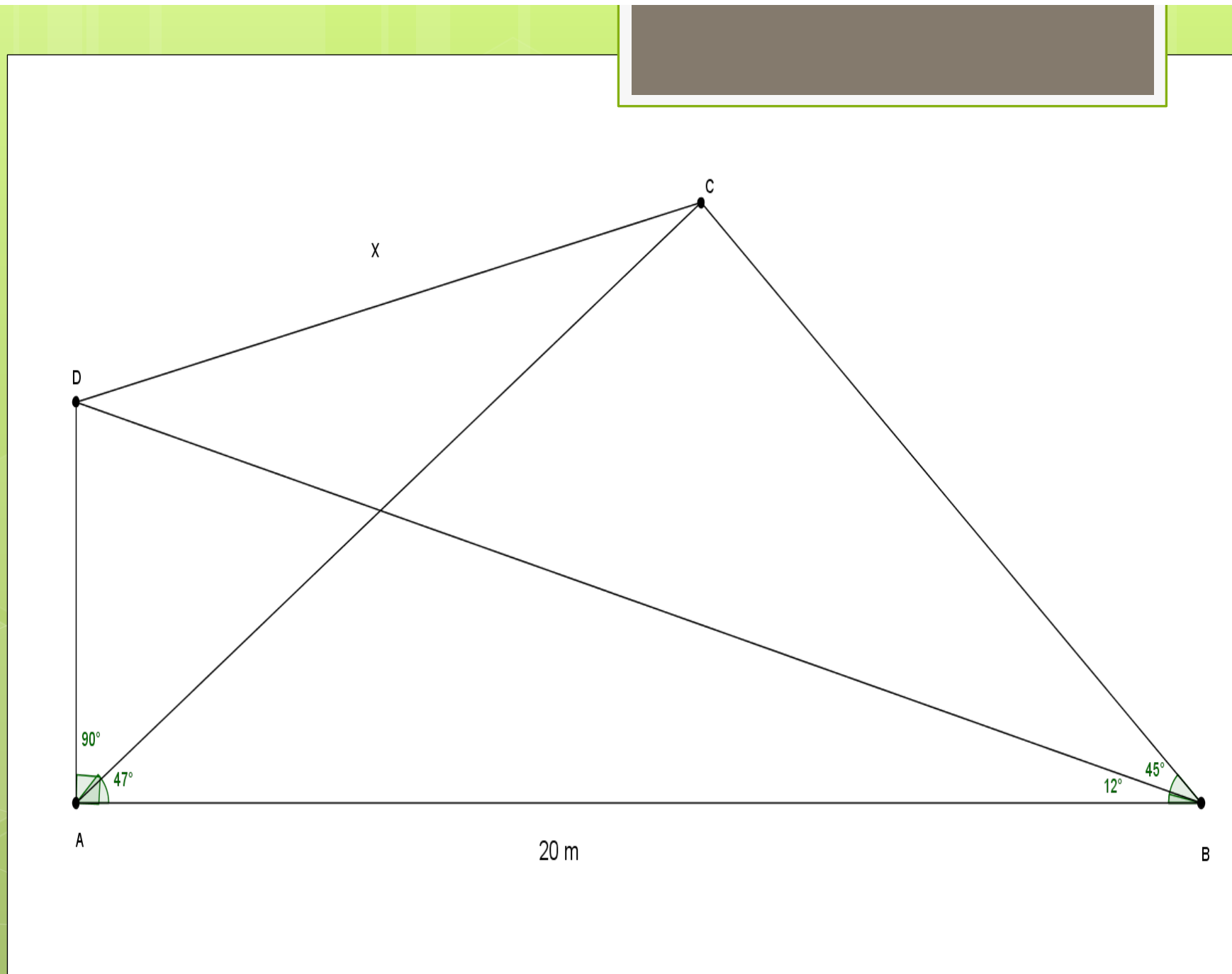






RJEŠENJA





$$20 : y = \sin(45^\circ + 47^\circ) : \sin 45^\circ$$

$$20 : z = \sin(90^\circ + 12^\circ) : \sin 12^\circ$$

$$y = \frac{20 \sin 45^\circ}{\sin(45^\circ + 47^\circ)} = 14,15 \text{ m}$$

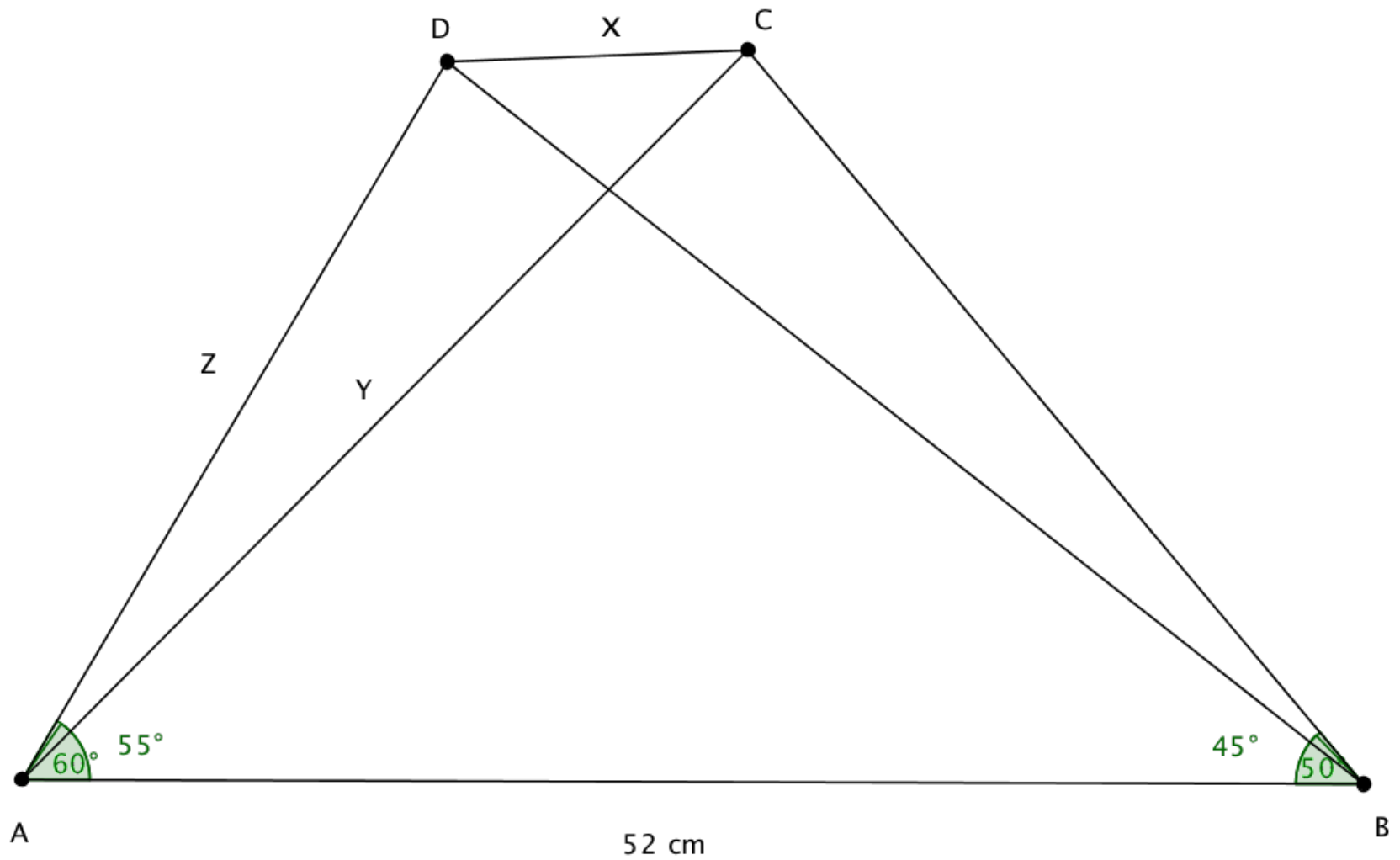
$$z = \frac{20 \sin 12^\circ}{\sin(90^\circ + 12^\circ)} = 4,25 \text{ m}$$

$$x^2 = z^2 + y^2 - 2 \cdot z \cdot y \cdot \cos(90^\circ - 47^\circ)$$

$$x^2 = 200,22 + 18.0625 - 87,964 = 130,3185$$

$$x = 11 \text{ m}$$





$$52 : y = \sin (55^\circ + 50^\circ) : \sin 50^\circ$$

$$52 : z = \sin (60^\circ + 45^\circ) : \sin 45^\circ$$

$$y = \frac{52 \sin 50^\circ}{\sin (55^\circ + 50^\circ)} = 41,24 \text{ cm}$$

$$z = \frac{52 \sin 45^\circ}{\sin (60^\circ + 45^\circ)} = 38,1 \text{ cm}$$

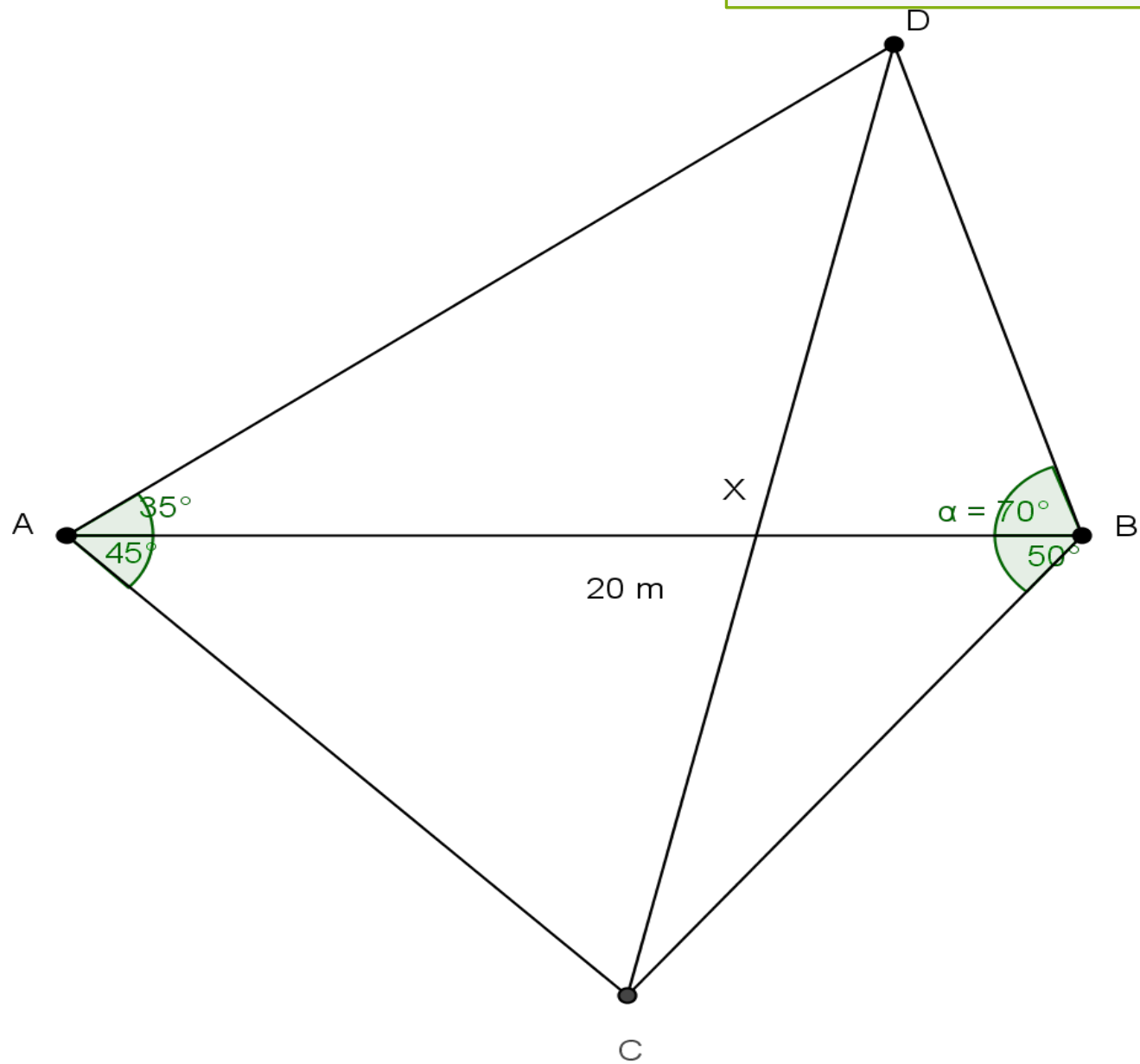
$$x^2 = z^2 + y^2 - 2 \cdot z \cdot y \cdot \cos (60^\circ - 55^\circ)$$

$$x^2 = 3152,35 - 2 \cdot 38,1 \cdot 41,24 \cdot 0,99619$$

$$x^2 = 21,834$$

$$x = 4,67 \text{ cm}$$





$$|AD| : 20 = \sin 70^\circ : \sin (35^\circ + 70^\circ)$$

$$|AC| : 20 = \sin 50^\circ : \sin (45^\circ + 50^\circ)$$

$$|AD| = \frac{20 \sin 70^\circ}{\sin (35^\circ + 70^\circ)} = 19,46 \text{ m}$$

$$|AC| = \frac{20 \sin 50^\circ}{\sin (45^\circ + 50^\circ)} = 15,38 \text{ m}$$

$$x^2 = |AD|^2 + |AC|^2 - 2 \cdot |AD| \cdot |AC| \cdot \cos 80^\circ$$

$$x^2 = 378,69 + 236,544 - 103,94 = 511,29$$

$$x = 22,61 \text{ m}$$