

DRŽAVNO NATJECANJE IZ LOGIKE 2024.

A KATEGORIJA

RJEŠENJA

Zadatak 1.

DA, NE

Ukupno 6 bodova.

Zadatak 2.

	Formula	(a)	(b)	(c)	(d)
i.	$\exists x \forall y (\neg P y x)$	I	N	N	I
ii.	$\forall x \exists y \forall z ((S z x \rightarrow \neg S z y) \wedge (\neg S z y \rightarrow S z x))$	N	I	N	N
iii.	$\exists x \exists y \exists z \exists u (\neg ((S x y \wedge S z u) \rightarrow \neg S y z))$	I	I	I	I
iv.	$\forall x \forall y \forall z ((B x \wedge B y \wedge \forall u ((P u x \vee P u y) \rightarrow P u z)) \rightarrow B z)$	N	N	N	N
v.	$\forall x ((\forall y (P y x)) \rightarrow B x)$	I	N	I	I
vi.	$\forall x \exists y (B y \wedge P y x \wedge \forall z ((B z \wedge P z x) \rightarrow P z y))$	N	I	N	N
vii.	$\forall x \forall y \forall z (P x z \vee \neg (P x y \vee P y x))$	N	I	N	N
viii.	$\forall x \forall y (\neg P x y \rightarrow (\neg P y x \rightarrow (\neg P x y \rightarrow P y x)))$	N	I	N	N
ix.	$\forall x (\neg \exists u S x u \vee \exists y \exists z (S x y \wedge S x z))$	I	I	I	I

Ukupno 72 boda.

Zadatak 3.

Dedukcija: prva varijanta

1	$\forall \odot \forall x \forall y \exists z x \odot y = z$	pretp.
2	$\forall x \forall y x \ell y = x$	pretp.
3	$\forall x \forall y (x \neg y = \top \leftrightarrow \neg x \ell y = \top)$	pretp.
4	$\forall x \forall y (x \vee y = \top \leftrightarrow (x \ell y = \top \vee y \ell x = \top))$	pretp.
5	$\forall \odot_1 \forall \odot_2 \forall \odot_3 \exists \odot_4 \forall x \forall y (x \odot_4 y = \top \leftrightarrow \exists u \exists v ((x \odot_1 y = u \wedge x \odot_2 y = v) \wedge u \odot_3 v = \top))$	pretp.
6	$\forall \odot_2 \forall \odot_3 \exists \odot_4 \forall x \forall y (x \odot_4 y = \top \leftrightarrow \exists u \exists v ((x \ell y = u \wedge x \odot_2 y = v) \wedge u \odot_3 v = \top))$	$\forall i, 5$
7	$\forall \odot_3 \exists \odot_4 \forall x \forall y (x \odot_4 y = \top \leftrightarrow \exists u \exists v ((x \ell y = u \wedge x \neg y = v) \wedge u \odot_3 v = \top))$	$\forall i, 6$
8	$\exists \odot_4 \forall x \forall y (x \odot_4 y = \top \leftrightarrow \exists u \exists v ((x \ell y = u \wedge x \neg y = v) \wedge u (\vee) v = \top))$	$\forall i, 7$
9	$\forall x \forall y (x \oplus y = \top \leftrightarrow \exists u \exists v ((x \ell y = u \wedge x \neg y = v) \wedge u (\vee) v = \top))$	pretp.
10	$\forall y (c \oplus y = \top \leftrightarrow \exists u \exists v ((c \ell y = u \wedge c \neg y = v) \wedge u (\vee) v = \top))$	$\forall i, 9$
11	$c \oplus d = \top \leftrightarrow \exists u \exists v ((c \ell d = u \wedge c \neg d = v) \wedge u (\vee) v = \top)$	$\forall i, 10$
12	$\neg (c \ell d = \top \vee \neg c \ell d = \top)$	pretp.
13	$c \ell d = \top$	pretp.
14	$c \ell d = \top \vee \neg c \ell d = \top$	$\vee u, 13$
15	\perp	$\perp u, 14, 12$
16	$\neg c \ell d = \top$	$\neg u, 13 - 15$
17	$c \ell d = \top \vee \neg c \ell d = \top$	$\vee u, 16$
18	\perp	$\perp u, 17, 12$
19	$\neg \neg (c \ell d = \top \vee \neg c \ell d = \top)$	$\neg u, 12 - 18$
20	$c \ell d = \top \vee \neg c \ell d = \top$	$\neg i, 19$
21	$c \ell d = \top$	pretp.
22	$\forall x \forall y \exists z x \neg y = z$	$\forall i, 1$
23	$\forall y \exists z c \neg y = z$	$\forall i, 22$
24	$\exists z c \neg d = z$	$\forall i, 23$
25	$c \neg d = f$	pretp.
26	$c \ell d = \top \wedge c \neg d = f$	$\wedge u, 21, 25$
27	$\forall y (\top \vee y = \top \leftrightarrow (\top \ell y = \top \vee y \ell \top = \top))$	$\forall i, 4$
28	$\top \vee f = \top \leftrightarrow (\top \ell f = \top \vee f \ell \top = \top)$	$\forall i, 27$
29	$\forall y \top \ell y = \top$	$\forall i, 2$
30	$\top \ell f = \top$	$\forall i, 29$
31	$\top \ell f = \top \vee f \ell \top = \top$	$\vee u, 30$
32	$\top \vee f = \top$	$\leftrightarrow i, 28, 31$
33	$(c \ell d = \top \wedge c \neg d = f) \wedge \top \vee f = \top$	$\wedge u, 26, 32$
34	$\exists v ((c \ell d = \top \wedge c \neg d = v) \wedge \top \vee v = \top)$	$\exists u, 33$
35	$\exists u \exists v ((c \ell d = u \wedge c \neg d = v) \wedge u (\vee) v = \top)$	$\exists u, 34$
36	$\exists u \exists v ((c \ell d = u \wedge c \neg d = v) \wedge u (\vee) v = \top)$	$\exists i, 24, 25 - 35$
37	$\neg c \ell d = \top$	pretp.
38	$\forall x \forall y \exists z x \ell y = z$	$\forall i, 1$
39	$\forall y \exists z c \ell y = z$	$\forall i, 38$
40	$\exists z c \ell d = z$	$\forall i, 39$
41	$c \ell d = g$	pretp.
42	$\forall y (c \neg y = \top \leftrightarrow \neg c \ell y = \top)$	$\forall i, 3$
43	$c \neg d = \top \leftrightarrow \neg c \ell d = \top$	$\forall i, 42$
44	$c \neg d = \top$	$\leftrightarrow i, 43, 37$
45	$c \ell d = g \wedge c \neg d = \top$	$\wedge u, 41, 44$
46	$\forall y (g \vee y = \top \leftrightarrow (g \ell y = \top \vee y \ell g = \top))$	$\forall i, 4$
47	$g \vee \top = \top \leftrightarrow (g \ell \top = \top \vee \top \ell g = \top)$	$\forall i, 46$
48	$\forall y \top \ell y = \top$	$\forall i, 2$
49	$\top \ell g = \top$	$\forall i, 48$
50	$g \ell \top = \top \vee \top \ell g = \top$	$\vee u, 49$
51	$g \vee \top = \top$	$\leftrightarrow i, 47, 50$
52	$(c \ell d = g \wedge c \neg d = \top) \wedge g \vee \top = \top$	$\wedge u, 45, 51$
53	$\exists v ((c \ell d = g \wedge c \neg d = v) \wedge g \vee v = \top)$	$\exists u, 52$
54	$\exists u \exists v ((c \ell d = u \wedge c \neg d = v) \wedge u (\vee) v = \top)$	$\exists u, 53$
55	$\exists u \exists v ((c \ell d = u \wedge c \neg d = v) \wedge u (\vee) v = \top)$	$\exists i, 40, 41 - 54$
56	$\exists u \exists v ((c \ell d = u \wedge c \neg d = v) \wedge u (\vee) v = \top)$	$\forall i, 20, 21 - 36, 37 - 55$
57	$c \oplus d = \top$	$\leftrightarrow i, 11, 56$
58	$\forall y c \oplus y = \top$	$\forall u, 57$
59	$\forall x \forall y x \oplus y = \top$	$\forall u, 58$
60	$\exists \odot \forall x \forall y x \odot y = \top$	$\exists u, 59$
61	$\exists \odot \forall x \forall y x \odot y = \top$	$\exists i, 8, 9 - 60$

Druga varijanta

1	$\forall \odot \forall x \forall y \exists z x \odot y = z$	pretp.
2	$\forall x \forall y x \odot y = x$	pretp.
3	$\forall x \forall y (x \neg y = \top \leftrightarrow \neg x \odot y = \top)$	pretp.
4	$\forall x \forall y (x \vee y = \top \leftrightarrow (x \odot y = \top \vee y \odot x = \top))$	pretp.
5	$\forall \odot_1 \forall \odot_2 \forall \odot_3 \exists \odot_4 \forall x \forall y (x \odot_4 y = \top \leftrightarrow \exists u \exists v ((x \odot_1 y = u \wedge x \odot_2 y = v) \wedge u \odot_3 v = \top))$	pretp.
6	$\exists y ((a \neg e = \top \leftrightarrow y \odot e = \top) \wedge (c \odot e = \top \leftrightarrow y \neg e = \top))$	pretp.
7	$(a \neg e = \top \leftrightarrow b \odot e = \top) \wedge (c \odot e = \top \leftrightarrow b \neg e = \top)$	pretp.
8	$a \neg e = \top \leftrightarrow b \odot e = \top$	$\wedge i, 7$
9	$c \odot e = \top \leftrightarrow b \neg e = \top$	$\wedge i, 7$
10	$\forall y (a \neg y = \top \leftrightarrow \neg a \odot y = \top)$	$\forall i, 3$
11	$a \neg e = \top \leftrightarrow \neg a \odot e = \top$	$\forall i, 10$
12	$\forall y (b \neg y = \top \leftrightarrow \neg b \odot y = \top)$	$\forall i, 3$
13	$b \neg e = \top \leftrightarrow \neg b \odot e = \top$	$\forall i, 12$
14	$a \odot e = \top$	pretp.
15	$b \odot e = \top$	pretp.
16	$a \neg e = \top$	$\leftrightarrow i, 8, 15$
17	$\neg a \odot e = \top$	$\leftrightarrow i, 11, 16$
18	\perp	$\perp u, 14, 17$
19	$\neg b \odot e = \top$	$\neg u, 15 - 18$
20	$b \neg e = \top$	$\leftrightarrow i, 13, 19$
21	$c \odot e = \top$	$\leftrightarrow i, 9, 20$
22	$c \odot e = \top$	pretp.
23	$\neg a \odot e = \top$	pretp.
24	$a \neg e = \top$	$\leftrightarrow i, 11, 23$
25	$b \neg e = \top$	$\leftrightarrow i, 9, 22$
26	$b \odot e = \top$	$\leftrightarrow i, 8, 24$
27	$\neg b \odot e = \top$	$\leftrightarrow i, 13, 25$
28	\perp	$\perp u, 26, 27$
29	$\neg \neg a \odot e = \top$	$\neg u, 23 - 28$
30	$a \odot e = \top$	$\neg i, 29$
31	$a \odot e = \top \leftrightarrow c \odot e = \top$	$\leftrightarrow u, 14 - 21, 22 - 30$
32	$a \odot e = \top \leftrightarrow c \odot e = \top$	$\exists i, 6, 7 - 31$
33	$\exists y ((a \neg e = \top \leftrightarrow y \odot e = \top) \wedge (c \odot e = \top \leftrightarrow y \neg e = \top)) \rightarrow (a \odot e = \top \leftrightarrow c \odot e = \top)$	$\rightarrow u, 6 - 32$
34	$\forall w (\exists y ((a \neg w = \top \leftrightarrow y \odot w = \top) \wedge (c \odot w = \top \leftrightarrow y \neg w = \top)) \rightarrow (a \odot w = \top \leftrightarrow c \odot w = \top))$	$\forall u, 33$
35	$\forall z \forall w (\exists y ((a \neg w = \top \leftrightarrow y \odot w = \top) \wedge (z \odot w = \top \leftrightarrow y \neg w = \top)) \rightarrow (a \odot w = \top \leftrightarrow z \odot w = \top))$	$\forall u, 34$
36	$\forall x \forall z \forall w (\exists y ((x \neg w = \top \leftrightarrow y \odot w = \top) \wedge (z \odot w = \top \leftrightarrow y \neg w = \top)) \rightarrow (x \odot w = \top \leftrightarrow z \odot w = \top))$	$\forall u, 35$

Treća varijanta

1	$\forall \odot \forall x \forall y \exists z x \odot y = z$	pretp.
2	$\forall x \forall y x \odot y = x$	pretp.
3	$\forall x \forall y (x \neg y = \top \leftrightarrow \neg x \odot y = \top)$	pretp.
4	$\forall x \forall y (x \vee y = \top \leftrightarrow (x \odot y = \top \vee y \odot x = \top))$	pretp.
5	$\forall \odot_1 \forall \odot_2 \forall \odot_3 \exists \odot_4 \forall x \forall y (x \odot_4 y = \top \leftrightarrow \exists u \exists v ((x \odot_1 y = u \wedge x \odot_2 y = v) \wedge u \odot_3 v = \top))$	pretp.
6	$\exists y \top \neg y = c$	pretp.
7	$\forall w d \odot w = \top$	pretp.
8	$\forall y (d \vee y = \top \leftrightarrow (d \odot y = \top \vee y \odot d = \top))$	$\forall i, 4$
9	$d \vee c = \top \leftrightarrow (d \odot c = \top \vee c \odot d = \top)$	$\forall i, 8$
10	$d \odot c = \top$	$\forall i, 7$
11	$d \odot c = \top \vee c \odot d = \top$	$\vee u, 10$
12	$d \vee c = \top$	$\leftrightarrow i, 9, 11$
13	$\forall w d \odot w = \top \rightarrow d \vee c = \top$	$\rightarrow u, 7 - 12$
14	$\forall y (\forall w y \odot w = \top \rightarrow y \vee c = \top)$	$\forall u, 13$
15	$\exists y \top \neg y = c \rightarrow \forall y (\forall w y \odot w = \top \rightarrow y \vee c = \top)$	$\rightarrow u, 6 - 14$
16	$\forall z (\exists y \top \neg y = z \rightarrow \forall y (\forall w y \odot w = \top \rightarrow y \vee z = \top))$	$\forall u, 15$

Ukupno (najviše) 96 bodova.